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Quantum Cloning of an Unknown Two-Particle Entangled State with Assistance

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We propose a protocol where one can realize quantum cloning of an unknown two-particle entangled state and its orthogonal-complement state with assistance offered by a state preparer. The first stage of the protocol requires usual teleportation using a (or two) four-particle entangled state(s) as quantum channel(s). In the second stage of the protocol, with the assistance (through a two-particle projective measurement) of the preparer, the perfect copies and complement copies of an unknown state can be produced.

KEY WORDS: quantum cloning; two-particle entangled state; two-particle projective measurement.

1. INTRODUCTION

Quantum entanglement has generated much interest in the quantum information processing such as quantum teleportation (Bennett *et al.*, 1993), quantum dense coding (Bennett and Wiesner, 1992), remote state preparation (Pati, 2001), and quantum key distribution (Ekert, 1991). In recent years, the possibility of cloning quantum states approximately has attracted much attention. A quantum state can not be cloned exactly because of the no-cloning theorem (Dieks, 1982; Wootters and Zerek, 1982). However, quantum cloning approximately is necessary in quantum information (Nielsen and Chuang, 2000). Though exact cloning is not possible, in the literature various cloning machines have been proposed which operate either in a deterministic or probabilistic way. Universal quantum cloning machines was originally addressed by Bužek and Hillery (1996). The deterministic state-dependent cloning machine, proposed firstly by Hillery and Bužek (1997), is designed to generate approximate clones of states belonging to a finite set. Gisin and Massar (1997), and Bruβ *et al.* (1998) constructed the universal qubit cloner

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that maximizes the local fidelity. The probabilistic cloning machine was first considered by Duan and Guo (1998) using a general unitary-reduction operation with a postelection of the measurement results. Murao *et al.* (1999) proposed the quantum telecloning process combining quantum teleportation and optimal quantum cloning from one input to M outputs. The other category of quantum cloning machines were developed by some authors (Bruβ and Macchiavello, 2001; Feng *et al.*, 2002; Qiu, 2002; Zou *et al.*, 2003).

Recently, Pati (2000) proposed a scheme where one can produce perfect copies and orthogonal-complement copies of an arbitrary unknown state with minimal assistance from a state preparer. This scheme realizes perfect cloning and complementing of an unknown state using resources such as entangled state, Bell-state measurement, single-particle von Neumann measurement, and classical communication. The purpose of this paper is to give a protocol that can produce perfect copies and orthogonal-complement copies of an unknown two-particle entangled state *via* a four-particle entangled state as the quantum channel. Different from the previous protocol using a single-particle von Nenmann orthogonal measurement (Pati, 2000), here we will realize the assisted cloning by using a two-particle projective measurement consisting of a set of nonmaximally entangled basis vectors. In addition, we also consider the assisted cloning *via* two four-particle entangled states as the quantum channels.

2. ASSISTED CLONING OF A TWO-PARTICLE ENTANGLED STATE BY A FOUR-PARTICLE GHZ STATE

Suppose Alice has an input two-particle entangled state $|\phi\rangle_{12} = \alpha|00\rangle_{12} + \beta|11\rangle_{12}$ from a state preperer Victor, with α as a real number and β as a complex number, and $|\alpha|^2 + |\beta|^2 = 1$. Assume Alice and Bob share a four-particle entangled state of the type from Greenberger–Horne–Zeilinger (GHZ) (Greenberger *et al.*, 1989) give by

$$|\phi\rangle_{3456} = \frac{1}{\sqrt{2}}(|0011\rangle_{3456} + |1100\rangle_{3456}). \tag{1}$$

Here, we assume that particles 3 and 4 belong to Alice, while particles 5 and 6 belong to Bob. The input state $|\phi\rangle_{12}$ is unknown to both Alice and Bob. The initial state of the combined system is

$$\begin{aligned} |\Psi\rangle &= |\phi\rangle_{12} \otimes |\phi\rangle_{3456} \\ &= |\Psi_{(1)}\rangle + |\Psi_{(2)}\rangle, \end{aligned} \tag{2}$$

where

$$|\Psi_{(1)}\rangle = \frac{1}{2\sqrt{2}}|\Phi^{\pm}\rangle_{13}|\Phi^{\pm}\rangle_{24}(\alpha|11\rangle \pm \pm\beta|00\rangle)_{56},$$

$$|\Psi_{(2)}\rangle = \frac{1}{2\sqrt{2}}|\Psi^{\pm}\rangle_{13}|\Psi^{\pm}\rangle_{24}(\alpha|00\rangle \pm \pm\beta|11\rangle)_{56}, \quad (3)$$

where $|\Phi^{\pm}\rangle_{ij}$ and $|\Psi^{\pm}\rangle_{ij}$ are the Bell states of particles i and j

$$|\Phi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{ij},$$
$$|\Psi^{\pm}\rangle_{ij} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{ij}. \quad (4)$$

In Equation (3), the notes "±" in the column from right to left correspond to the Bell state of particles (1,3) and (2,4), respectively. Assume Alice performs Bell-state measurements on particles (1,3) and (2,4), respectively, and if the measurement outcome of Alice is $|\Psi^{+}\rangle_{13}|\Psi^{-}\rangle_{24}$ (the probability of this result is only 1/8), then the resulting six-particle state can be written as

$$|\Psi^{-}\rangle_{24}\langle\Psi^{-}|\Psi^{+}\rangle_{13}\langle\Psi^{+}|\Psi\rangle = \frac{1}{2\sqrt{2}}|\Psi^{+}\rangle_{13}|\Psi^{-}\rangle_{24}(\alpha|00\rangle - \beta|11\rangle)_{56}. \quad (5)$$

After these measurements, Alice sends the measurement result to Bob through a classical channel. According to the measure outcome of Alice, Bob will operate a unitary transformation $I_5 \otimes (\sigma_z)_6$ on Equation (5), and to get the original state from particles 5 and 6.

To create either a copy or an orthogonal-complement copy of the unknown two-particle state $|\phi\rangle$, Alice needs assistance of Victor. According to the projection postulate of quantum mechanics, if Alice applies projectors $|\Psi^{-}\rangle_{24}\langle\Psi^{-}|\Psi^{+}\rangle_{13}\langle\Psi^{+}|$ into the combined state $|\Psi\rangle$, the state of particles 1, 2, 3, and 4 will collapse in the entangled state $|\Psi^{+}\rangle_{13}|\Psi^{-}\rangle_{24}$ (see Equation (5)). Alice sends particles 1 and 2 to Victor and keeps particles 3 and 4 in her possession. Since Victor knows the parameters α and β of original state $|\phi\rangle_{12}$ completely, he carries out a two-particle projective measurement on the particles 1 and 2 in a set of mutually orthogonal basis vectors $\{|\varphi\rangle, |\varphi_{\perp}\rangle, |\psi\rangle, |\psi_{\perp}\rangle\}$, which is given by

$$|\varphi\rangle_{12} = \alpha|00\rangle_{12} + \beta|11\rangle_{12},$$
$$|\varphi_{\perp}\rangle_{12} = \beta^{*}|00\rangle_{12} - \alpha|11\rangle_{12},$$
$$|\psi\rangle_{12} = \alpha|01\rangle_{12} + \beta|10\rangle_{12}, \quad (6)$$
$$|\psi_{\perp}\rangle_{12} = \beta^{*}|01\rangle_{12} - \alpha|10\rangle_{12}.$$

The earlier four nonmaximally entangled basis states $\{|\varphi\rangle, |\varphi_{\perp}\rangle, |\psi\rangle, |\psi_{\perp}\rangle\}$ are related to the computation basis vectors $\{\langle 00|, |01\rangle, |10\rangle, |11\rangle\}$, and form a complete orthogonal basis in a four-dimensional Hilbert space. We find that the $|\varphi\rangle_{12}$ is equal to $|\phi\rangle_{12}$ and the basis $|\varphi_{\perp}\rangle_{12}$ is equal to $|\phi_{\perp}\rangle_{12}$, where $|\phi_{\perp}\rangle_{12} = \beta^{*}|00\rangle_{12} - \alpha|11\rangle_{12}$ is the orthogonal-complement state to $|\phi\rangle_{12}$. Moreover,

$|\psi\rangle_{12} = (I_1 \otimes (\sigma_x)_2)|\phi\rangle_{12}$ and $|\psi_\perp\rangle_{12} = (I_1 \otimes (\sigma_x)_2)|\phi_\perp\rangle_{12}$ is the orthogonal-complement state to $|\psi\rangle_{12}$ (the σ_z and σ_x given earlier are Pauli operators). Thus, the entangled state $|\Psi^+\rangle_{13}|\Psi^-\rangle_{24}$ in the basis $\{|\varphi\rangle, |\varphi_\perp\rangle, |\psi\rangle, |\psi_\perp\rangle\}$ can be rewritten as

$$|\Psi^+\rangle_{13}|\Psi^-\rangle_{24} = \frac{1}{2}[|\varphi\rangle_{12}(\alpha|11\rangle_{34} - \beta^*|00\rangle_{34}) + |\varphi_\perp\rangle_{12}(\alpha|00\rangle_{34} + \beta|11\rangle_{34})$$
$$|\psi\rangle_{12}(-\alpha|10\rangle_{34} + \beta^*|01\rangle_{34}) + |\psi_\perp\rangle_{12}(\alpha|01\rangle_{34} + \beta|10\rangle_{34})]. \tag{7}$$

If the result of Victor's measurement on the two particle 1 and 2 is $|\varphi_\perp\rangle_{12}$, Equation (5) can be written as

$$|\varphi_\perp\rangle_{12}\langle\varphi_\perp|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Psi\rangle$$
$$= \frac{1}{4\sqrt{2}}|\varphi_\perp\rangle_{12} \otimes |\phi\rangle_{34} \otimes (I_5 \otimes (\sigma_z)_6)|\phi\rangle_{56}. \tag{8}$$

Victor sends the measurement outcome to Alice through a classical channel with two classical bits, then Alice knows that the state of her particles 3 and 4 has been found in the original state $|\phi\rangle_{34} = \alpha|00\rangle_{34} + \beta|11\rangle_{35}$, which is just a copy of the state $|\phi\rangle_{12}$. If the result of Victor is $|\varphi\rangle_{12}$, then two cbits from Victor to Alice would yield a complement state given by

$$|\varphi\rangle_{12}\langle\varphi|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Psi\rangle$$
$$= \frac{1}{4\sqrt{2}}|\varphi\rangle_{12} \otimes |\phi_\perp\rangle_{34} \otimes (I_5 \otimes (\sigma_z)_6)|\phi\rangle_{56}. \tag{9}$$

It is clear from Equation (9) that Alice gets a complement copy of the unknown state. From Equation (7), if the results of Victor are $|\psi\rangle_{12}$ and $|\psi_\perp\rangle_{12}$, Equation (5) can be written as, respectively

$$|\psi\rangle_{12}\langle\psi|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Psi\rangle$$
$$= \frac{1}{4\sqrt{2}}|\psi\rangle_{12} \otimes (I_3 \otimes (\sigma_x)_4)|\phi_\perp\rangle_{34} \otimes (I_5 \otimes (\sigma_z)_6)|\phi\rangle_{56}, \tag{10}$$

$$|\psi_\perp\rangle_{12}\langle\psi_\perp|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Psi\rangle$$
$$= -\frac{1}{4\sqrt{2}}|\psi_\perp\rangle_{12} \otimes (I_3 \otimes (\sigma_x)_4)|\phi\rangle_{34} \otimes (I_5 \otimes (\sigma_z)_6)|\phi\rangle_{56}. \tag{11}$$

From Equations (10) and (11), one can see that Alice gets a copy and a complement copy (all are up to doing a rotation operation) of the unknown state, respectively.

In the process of teleportation, if the Alice's measurement outcomes are other seven entangled states $|\Phi^\pm\rangle_{13}|\Phi^\pm\rangle_{24}$, $|\Psi^\pm\rangle_{13}|\Psi^+\rangle_{24}$, and $|\Psi^-\rangle_{13}|\Psi^-\rangle_{24}$, applying the same analysis method as the one mentioned earlier, Alice will obtain a copy (or complement copy) of the unknown state at her place.

3. ASSISTED CLONING USING TWO FOUR-PARTICLE ENTANGLED GHZ STATES

Now, we will generalize the previous scheme for producing more copies or complement copies using two four-particle GHZ states. Suppose that the unknown input state of Alice from Victor is still the two-particle entangled state $|\phi\rangle_{12} = \alpha|00\rangle_{12} + \beta|11\rangle_{12}$. The two four-particle GHZ states, as the quantum channels, are given by

$$|\phi\rangle_{3456} = \frac{1}{\sqrt{2}}(|0011\rangle + |1100\rangle)_{3456},$$

$$|\phi\rangle_{789,10} = \frac{1}{\sqrt{2}}(|0011\rangle + |1100\rangle)_{789,10}. \qquad (12)$$

Here Alice possesses particles 3 and 4, Bob possesses particles 5, 7, 8 and 9, and Carla possesses particles 6 and 10. The input state $|\phi\rangle_{12}$ is unknown to Alice, Bob, and Carla. The combined 10-particle state is expressed as

$$\begin{aligned}|\Phi\rangle &= |\phi\rangle_{12} \otimes |\phi\rangle_{3456} \otimes |\phi\rangle_{789,10} \\ &= |\Phi_{(1)}\rangle + |\Phi_{(2)}\rangle,\end{aligned} \qquad (13)$$

where

$$|\Phi_{(1)}\rangle = \frac{1}{4}|\Phi^{\pm}\rangle_{13}|\Phi^{\pm}\rangle_{24}$$
$$(\alpha|110011\rangle + \alpha|111100\rangle \pm \pm\beta|000011\rangle \pm \pm\beta|001100\rangle)_{56789,10}, \qquad (14)$$

$$|\Phi_{(2)}\rangle = \frac{1}{4}|\Psi^{\pm}\rangle_{13}|\Psi^{\pm}\rangle_{24}$$
$$(\alpha|000011\rangle + \alpha|001100\rangle \pm \pm\beta|110011\rangle \pm \pm\beta|111100\rangle)_{56789,10} \qquad (15)$$

In Equations (14) and (15), the notes "±" in the column from right to left correspond to the Bell state of particles (1,3) and (2,4), respectively.

Now let Alice carries out Bell-state measurements on her particles (1,3) and (2,4), respectively. If the result of Alice's measurement is $|\Psi^{+}\rangle_{13}|\Psi^{-}\rangle_{24}$ (the probability of this result is only 1/8), then the resulting state will be

$$|\Psi^{-}\rangle_{24}\langle\Psi^{-}|\Psi^{+}\rangle_{13}\langle\Psi^{+}|\Phi\rangle = \frac{1}{4}|\Psi^{+}\rangle_{13}|\Psi^{-}\rangle_{24}(\alpha|000011\rangle + \alpha|001100\rangle$$
$$- \beta|110011\rangle - \beta|111100\rangle)_{56789,10} = |\Phi'_{(1)}\rangle + |\Phi'_{(2)}\rangle, \qquad (16)$$

where

$$\left|\Phi'_{(1)}\right\rangle = \frac{1}{8}|\Psi^+\rangle_{13}|\Psi^-\rangle_{24}|\Phi^{\pm}\rangle_{57}|\Psi^{\pm}\rangle_{89}(\alpha|01\rangle - \pm\pm\beta|10\rangle)_{6,10}, \tag{17}$$

$$\left|\Phi'_{(2)}\right\rangle = \frac{1}{8}|\Psi^+\rangle_{13}|\Psi^-\rangle_{24}|\Psi^{\pm}\rangle_{57}|\Psi^{\pm}\rangle_{89}(\pm\alpha|00\rangle - \pm\beta|11\rangle)_{6,10}. \tag{18}$$

In Equations (17) and (18), the notes "±" in the column from right to left correspond to the Bell state of particles (5,7;8,9) and (8,9;5,7), respectively. After the measurements given earlier, Alice sends her resule *via* a classical channel with four bits of information to both Bob and Carla. In the next step, Bob performs another Bell-state measurements on his particles (5,7) and (8,9), respectively. If the result of Bob's measurement is $|\Psi^-\rangle_{57}|\Psi^+\rangle_{89}$, the resulting state (the probability of this result is only 1/64) will be written as

$$\begin{aligned} &|\Psi^+\rangle_{89}\langle\Psi^+|\Psi^-\rangle_{57}\langle\Psi^-|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Phi\rangle \\ &= \frac{1}{8}|\Psi^+\rangle_{13}|\Psi^-\rangle_{24}|\Psi^-\rangle_{57}|\Psi^+\rangle_{89}|\phi\rangle_{6,10}. \end{aligned} \tag{19}$$

From Equation (19), one can see that the state of particles 6 and 10 of Carla is found to be in the original state.

In the second stage of our protocol, Alice and Bob send particles (1,2) and (5,8), respectively, to Victor. When Victor gets the particles (1,2) and (5,8), he chooses to measure the states in the basis $\{|\varphi\rangle_{ij}, |\varphi_\perp\rangle_{ij}, |\psi\rangle_{ij}, |\psi_\perp\rangle_{ij}\}$, which is given by

$$\begin{aligned} |\varphi\rangle_{ij} &= \alpha|00\rangle_{ij} + \beta|11\rangle_{ij}, \\ |\varphi_\perp\rangle_{ij} &= \beta^*|00\rangle_{ij} - \alpha|11\rangle_{ij}, \\ |\psi\rangle_{ij} &= \alpha|01\rangle_{ij} + \beta|10\rangle_{ij}, \\ |\psi_\perp\rangle_{ij} &= \beta^*|01\rangle_{ij} - \alpha|10\rangle_{ij}, \end{aligned} \tag{20}$$

where $i, j = 1, 2$ or $5, 8$. In the new basis, the total state can be written as

$$\begin{aligned} &|\Psi^+\rangle_{89}\langle\Psi^+|\Psi^-\rangle_{57}\langle\Psi^-|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Phi\rangle \\ &= \tfrac{1}{32}[|\varphi\rangle_{12}(\alpha|11\rangle_{34} - \beta^*|00\rangle_{34}) + |\varphi_\perp\rangle_{12}(\alpha|00\rangle_{34} + \beta|11\rangle_{34}) \\ &\quad + |\psi\rangle_{12}(-\alpha|10\rangle_{34} + \beta^*|01\rangle_{34}) - |\psi_\perp\rangle_{12}(\alpha|01\rangle_{34} + \beta|10\rangle_{34})] \\ &\quad \times [|\varphi\rangle_{58}(\alpha|11\rangle_{79} - \beta^*|00\rangle_{79}) + |\varphi_\perp\rangle_{58}(\alpha|00\rangle_{79} + \beta|11\rangle_{79}) \\ &\quad + |\psi\rangle_{58}(\alpha|10\rangle_{79} - \beta^*|01\rangle_{79}) + |\psi_\perp\rangle_{58}(\alpha|01\rangle_{79} + \beta|10\rangle_{79})]|\phi\rangle_{6,10}. \end{aligned} \tag{21}$$

Assume Victor first carries out a two-particle projective measurement on the particles (1,2) and then on particles (5,8) and in both cases let the results be $|\varphi_\perp\rangle_{12}$ and $|\varphi_\perp\rangle_{58}$. Then Victor sends the classical information (two bits) to Alice and (two bits) to Bob. According to the information of Victor, Alice and Bob can find that their particles (3,4) and (7,9) are in the unknown state, respectively. Thus, the final state after two-particle projective measurements is given by

$$|\varphi_\perp\rangle_{12}\langle\varphi_\perp|\varphi_\perp\rangle_{58}\langle\varphi_\perp|\Psi^+\rangle_{89}\langle\Psi^+|\Psi^-\rangle_{57}\langle\Psi^-|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Phi\rangle$$
$$= \frac{1}{32}|\varphi_\perp\rangle_{12} \otimes |\phi\rangle_{34} \otimes |\varphi_\perp\rangle_{58} \otimes |\phi\rangle_{79} \otimes |\phi\rangle_{6,10}. \quad (22)$$

From Equation (22), it is clear that Alice, Bob, and Carla each get a perfect copy of the unknown state. If Victor's outcome for particles (1,2) and (5,8) are $|\psi\rangle_{12}$ and $|\psi\rangle_{58}$, the final state is given by

$$|\psi\rangle_{12}\langle\psi|\psi\rangle_{58}\langle\psi|\Psi^+\rangle_{89}\langle\Psi^+|\Psi^-\rangle_{57}\langle\Psi^-|\Psi^-\rangle_{24}\langle\Psi^-|\Psi^+\rangle_{13}\langle\Psi^+|\Phi\rangle$$
$$= -\frac{1}{32}|\psi\rangle_{12} \otimes (I_3 \otimes (\sigma_x)_4)|\phi_\perp\rangle_{34} \otimes |\psi\rangle_{58} \otimes (I_7 \otimes (\sigma_x)_9)|\phi_\perp\rangle_{79} \otimes |\phi\rangle_{6,10}. \quad (23)$$

After sending two classical bits to Alice and two to Bob from Victor, Alice knows that her state of particles 3 and 4 has been found in the complement copy of the unknown state (up to a rotation operator), and Bob gets a complement copy for his particles 7 and 9 (up to a rotation operator, too), and Carla gets the copy of the unknown state. If what Victor measure is another outcome for particles (1,2) and (5,8), by Equation (21) Alice and Bob can acquire a perfect copy or a complement copy (up to a rotation operator) of the unknown state.

4. CONCLUSION

We have proposed a protocol that can produce perfect copies or orthogonal-complement copies of an arbitrary unknown two-particle entangled state, *via* quantum and classical channel, with assistance. Our protocol requires resources such as a four-particle GHZ state (or two four-particle GHZ states) as quantum channel(s), Bell-state measurement, classical communication, and two-particle projective measurement. This protocol includes two stages. The first stage of the protocol requires usual teleportation. In the second stage, Victor (a preparer of state) will perform two-particle projective measurements on particles which from Alice and Bob. According to information from Victor, Alice and Bob can acquire either a perfect copy or an orthogonal-complement copy of unknown state.

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